Function

2019 M/J,O/N

0606/21/M/J/19

1. a) Sketch the graph of $y = |5x - 3|$, showing the coordinates of the points where the graph meets the coordinate axes.

[3]

$y = 5x - 3$

let $x = 0$, $y = 5(0) - 3 = 0 - 3 = -3$

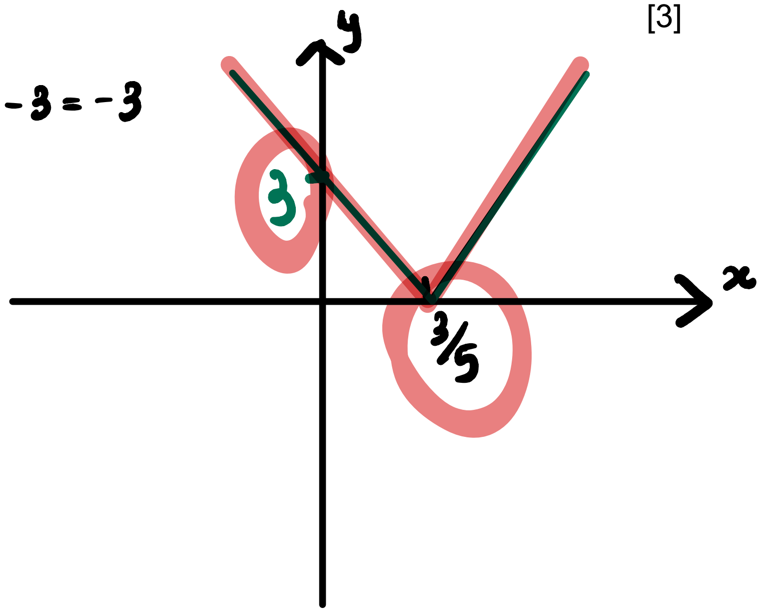
$y = 0$, $0 = 5x - 3$

$3 = 5x$

$x = \frac{3}{5}$

$(0, -3)$

$(\frac{3}{5}, 0)$



b) Solve the equation $|5x - 3| = 2 - x$.

[3]

$5x - 3 = 2 - x$

$5x + x = 2 + 3$

$6x = 5$

$x = \frac{5}{6}$

or

$5x - 3 = -2 + x$

$5x - x = -2 + 3$

$4x = 1$

$x = \frac{1}{4}$

#

$(x+q)(x+q) = x^2 + 2qx + q^2$

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2. (i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

$5x^2 - 15x + 1 = p(x+q)^2 + r$

$= p(x^2 + 2qx + q^2) + r$

$= px^2 + 2pqx + pq^2 + r$

$\therefore p = 5$

$2pq = -15$

$10q = -15$

$q = -3/2$

$pq^2 + r = 1$

$5 \times \frac{9}{4} + r = 1$

$\frac{45}{4} + r = 1$

$r = -\frac{41}{4}$

$5x^2 - 15x + 1 = 5\left(x - \frac{3}{2}\right)^2 - \frac{41}{4}$

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

$\min\left(\frac{3}{2}, -\frac{41}{4}\right)$

$-41/4 \div 5 = \frac{-41}{20}$, $x = \frac{3}{2}$

3. (a) The functions f and g are defined by

$f(x) = 5x - 2$ for $x > 1$,

$g(x) = 4x^2 - 9$ for $x > 0$

(i) State the range of g. [1]

$g(x) = 4(0)^2 - 9$

$= -9$ $y > -9$

(ii) Find the domain of gf. [1]

$x > 1$

$f(x) = g^{-1}(y)$

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

$gf(x) = g(5x-2)$

$= 4(5x-2)^2 - 9$

$4(5x-2)^2 - 9 = 4$

$4(5x-2)^2 = 13$

$(5x-2)^2 = \frac{13}{4}$

$5x - 2 = \sqrt{\frac{13}{4}}$

$5x = \frac{\sqrt{13}}{2} + 2$

$x = \frac{\sqrt{13}}{10} + \frac{2}{5} = \frac{\sqrt{13}+4}{10}$

(b) The function h is defined by $h(x) = \sqrt{x^2 - 1}$ for $x \le -1$.

(i) State the geometrical relationship between the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [1]

reflection in $x = y$

$h(x) = \sqrt{x^2-1}\ ,\ x \leq -1$

$y = \sqrt{x^2-1}$

$x = \sqrt{y^2-1}$

$x^2 = y^2-1$

$x^2+1 = y^2$

$\pm\sqrt{x^2+1} = y$

(ii) Find an expression for $h^{-1}(x)$. [3]

$h^{-1}(x) = -\sqrt{x^2+1}$ ✱

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4. It is given that $f: x \to \sqrt{x}$ for $x \geq 0$,
$g: x \to x + 5$ for $x \geq 0$

$f^{-1}(x) = x^2$

$g^{-1}(x) = x-5$

Identify each of the following functions with one of $f^{-1}, g^{-1}, fg, gf, f^2, g^2$.

(i) $\sqrt{x + 5}$ [1]

$fg(x)$

(ii) $x - 5$ [1]

$g^{-1}(x)$

(iii) x^2 [1]

$f^{-1}(x)$

(iv) $x + 10$ [1]

$gg(x)$

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5. (i) Draw the graph of $y = |2x - 3|$. [2]

$y=0,\ 0 = 2x-3$

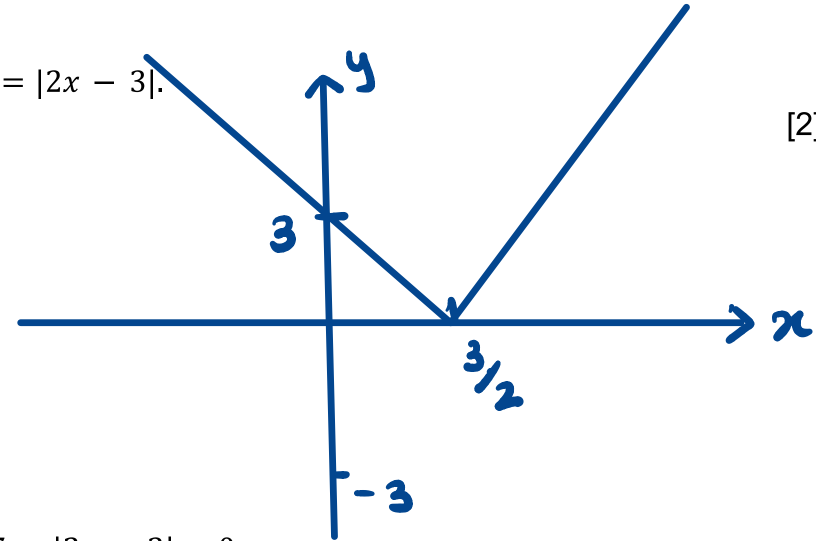
$x = \frac{3}{2}$

$x=0,$

$y = -3$

$(\frac{3}{2}, 0)$

$(0, -3)$



(ii) Solve the equation $7 - |2x - 3| = 0$. [3]

$-|2x-3| = -7$

$|2x-3| = 7$

$2x - 3 = 7$

$2x = 10$

$x = 5$

or

$2x - 3 = -7$

$2x = -4$

$x = -2$

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6. Solve $|3x + 2| = x + 4$. [3]

$3x+2 = x+4$

$3x - x = 4-2$

$2x = 2$

$x = 1$

or

$3x+2 = -x-4$

$3x + x = -4-2$

$4x = -6$

$x = -\frac{3}{2}$

7. (i) Given that $y = 2x^2 - 4x - 7$, write y in the form $a(x - b)^2 + c$, where a, b and c are constants. [3]

$$2x^2 - 4x - 7 = a(x - b)^2 + c$$
$$= a(x^2 - 2bx + b^2) + c$$
$$= ax^2 - 2abx + ab^2 + c$$

$2x^2 = ax^2$

$a = 2$

$\cancel{-}4\cancel{x} = \cancel{-}2ab\cancel{x}$

$4 = 4b$

$b = 1$

$ab^2 + c = -7$

$2 + c = -7$

$c = -9$

$2x^2 - 4x - 7 = 2(x-1)^2 - 9$ ✗

(ii) Hence write down the minimum value of y and the value of x at which it occurs. [2]

stationary pt $= (1, -9)$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

minimum value of $y = -9$

value of $x = 1$